

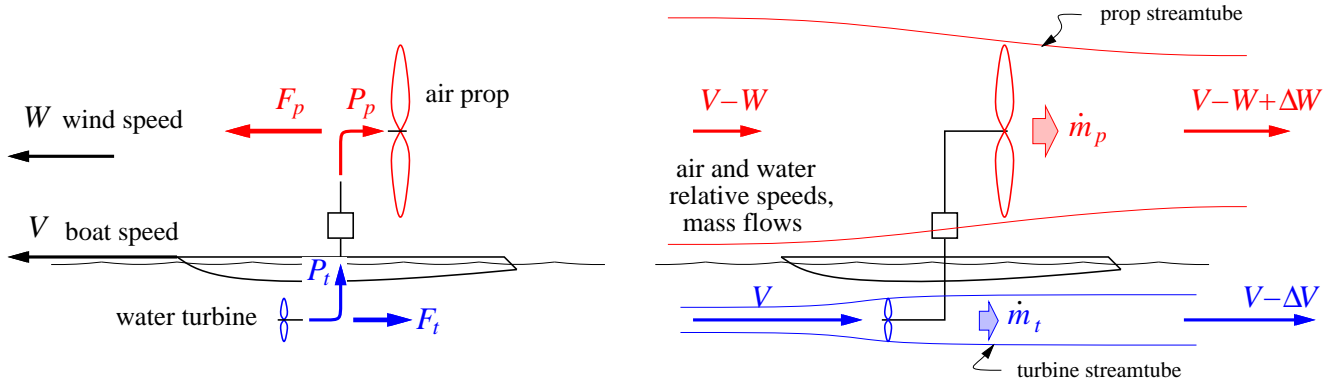
# DDWFTTW Power Analysis

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## Nomenclature

$V$ boat speed	$W$ wind speed
$\Delta V$ water turbine velocity change	$\Delta W$ air prop velocity change
$\dot{m}_t$ water turbine mass flow	$\dot{m}_p$ air prop mass flow
$F_t$ drag force on water turbine	$F_p$ thrust force on air prop
$P_t$ shaft power out of water turbine	$P_p$ shaft power into air prop



## Velocities

The figure above shows a boat moving with water-speed  $V$ , in the same direction as a slower wind speed  $W$ . The water turbine therefore sees a water velocity of  $V$ , while the air prop sees an air velocity of  $V - W$ , both opposite the boat motion. The downstream velocity changes in the prop and turbine streamtubes are  $\Delta W$  and  $\Delta V$ .

## Force–Momentum Relations

The force on each rotor is equal to the rate of axial momentum change in each streamtube.

$$F_p = \dot{m}_p \Delta [v_p] = \dot{m}_p [(V - W + \Delta W) - (V - W)] = \dot{m}_p \Delta W \quad (1)$$

$$F_t = -\dot{m}_t \Delta [v_t] = -\dot{m}_t [(V - \Delta V) - V] = \dot{m}_t \Delta V \quad (2)$$

The negative sign for  $F_t$  is included because  $F_t$  defined positive as shown in the figure, which corresponds to a negative  $\Delta [v_{\text{water}}]$ .

## Power–Kinetic Energy Relations

The shaft power of each rotor is equal to the rate of kinetic energy change in each streamtube.

$$P_p = \frac{1}{2} \dot{m}_p \Delta [v_p^2] = \frac{1}{2} \dot{m}_p [(V - W + \Delta W)^2 - (V - W)^2] = \dot{m}_p [(V - W)\Delta W + \frac{1}{2}\Delta W^2] \quad (3)$$

$$P_t = -\frac{1}{2} \dot{m}_t \Delta [v_t^2] = -\frac{1}{2} \dot{m}_t [(V - \Delta V)^2 - V^2] = \dot{m}_t [V\Delta V - \frac{1}{2}\Delta V^2] \quad (4)$$

The negative sign for  $P_t$  is included for the same reason as for  $F_t$ . Using the previous force relations, the power relations can also be given as follows.

$$P_p = F_p [(V - W) + \frac{1}{2}\Delta W] \quad (5)$$

$$P_t = F_t [V - \frac{1}{2}\Delta V] \quad (6)$$

## Net Power

The net power available from the prop and turbine combination is

$$P_{\text{net}} = P_t - P_p = F_p W + (F_t - F_p)V - \frac{1}{2}F_t \Delta V - \frac{1}{2}\Delta W \quad (7)$$

If the vehicle is in steady-state operation, the net thrust must be equal to the net drag.

$$F_{\text{net}} = F_p - F_t = D \quad (8)$$

The net power then becomes

$$P_{\text{net}} = P_t - P_p = F_p W - DV - \frac{1}{2}F_t \Delta V - \frac{1}{2}F_p \Delta W \quad (9)$$

The four contributions to the net power (9) can now be readily interpreted:

$F_p W$	power produced by prop thrust moving at wind speed
$-DV$	power lost to drag force moving at boat speed
$-\frac{1}{2}F_t \Delta V$	power lost due to kinetic energy deposited in the water
$-\frac{1}{2}F_p \Delta W$	power lost due to kinetic energy deposited in the air

In steady-state operation,  $P_{\text{net}}$  must be sufficiently positive to balance the remaining power losses in the system. The main remaining losses not accounted for are power-transmission losses, profile-drag losses on the prop and turbine blades, and swirl losses in the prop and turbine slipstreams.