THE ANCIENT INTERFACE

FASTER THAN THE WIND

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Orange, California

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INTRODUCTION

If one is given a uniform wind traveling over a level surface, is it possible to construct a man-carrying vehicle which by use of wind energy alone can accelerate in the wind direction from zero speed up to a speed larger than the wind speed? For variety, the level surface may be considered to be either a hard surface or a body of water. The work described in this paper has been carried out for the purpose of answering this problem.

Clearly, no ordinary sailboat can perform this task, inasmuch as the relative wind drops to zero as the boat approaches the wind velocity, whereas the hull drag is constantly increasing. A land vehicle has essentially the same problem because of rolling friction. Therefore, one is tempted to answer this problem in the negative.

The above problem had its genesis at the University of Michigan where it was communicated to Mr. D.L. Elder some 20 years ago. The proposed solution involved the use of a propeller geared to the wheels in the case of a land vehicle or geared to a second propeller in the water for the boating case. Thus, the usual sail was to be replaced by a propeller geared to a wheel or second propeller. If sufficient power were available to drive the first propeller, the vehicle could travel faster than the wind, but as soon as one suggests that this power be obtained by gearing the propeller to either the wheels or the water propeller, most individuals

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will associate the idea with perpetual motion and will claim that the vehicle cannot possibly travel faster than the wind.

The writer has always claimed that such a vehicle can be built and operated successfully. The technical part of this paper serves to analyze the performance of such a vehicle, and performance charts for typical water and land vehicles are presented. A second part describes the land vehicle which has been built by the writer and operated at speeds faster than the wind.

LAND VEHICLE PERFORMANCE

Because the land vehicle is somewhat simpler than the boating case, the land vehicle will be discussed first. A schematic diagram is shown in Figure 1. The wind speed is denoted by V_W and the vehicle velocity by V_c. The airscrew or propeller of radius R is geared to the wheels such that the propeller rotational speed Ω is given by

$$\Omega = \frac{GV_c}{R} \tag{1}$$

so that the propeller tip is travelling forward at speed V_c and in the propeller plane at speed GV_c. For convenience, when G and V_c are positive the propeller shaft rotation Ω as viewed from behind the vehicle will be taken to be clockwise. The thrust on the propeller is denoted by T_1; the force T_2 on the vehicle is that part of the wheel reaction with the roadway as a result of the aerodynamic torque transmitted through the propeller shaft and gearing. The vehicle weight is denoted by W, and μW is the force on the vehicle equal to

$$\mu W = W(\mu_1 + \mu_2 + \mu_3) \tag{2}$$

where μ_1 is the result of the mechanical rolling friction, μ_2 represents the aerodynamic friction on the vehicle body, and $\mu_3 W$ is the

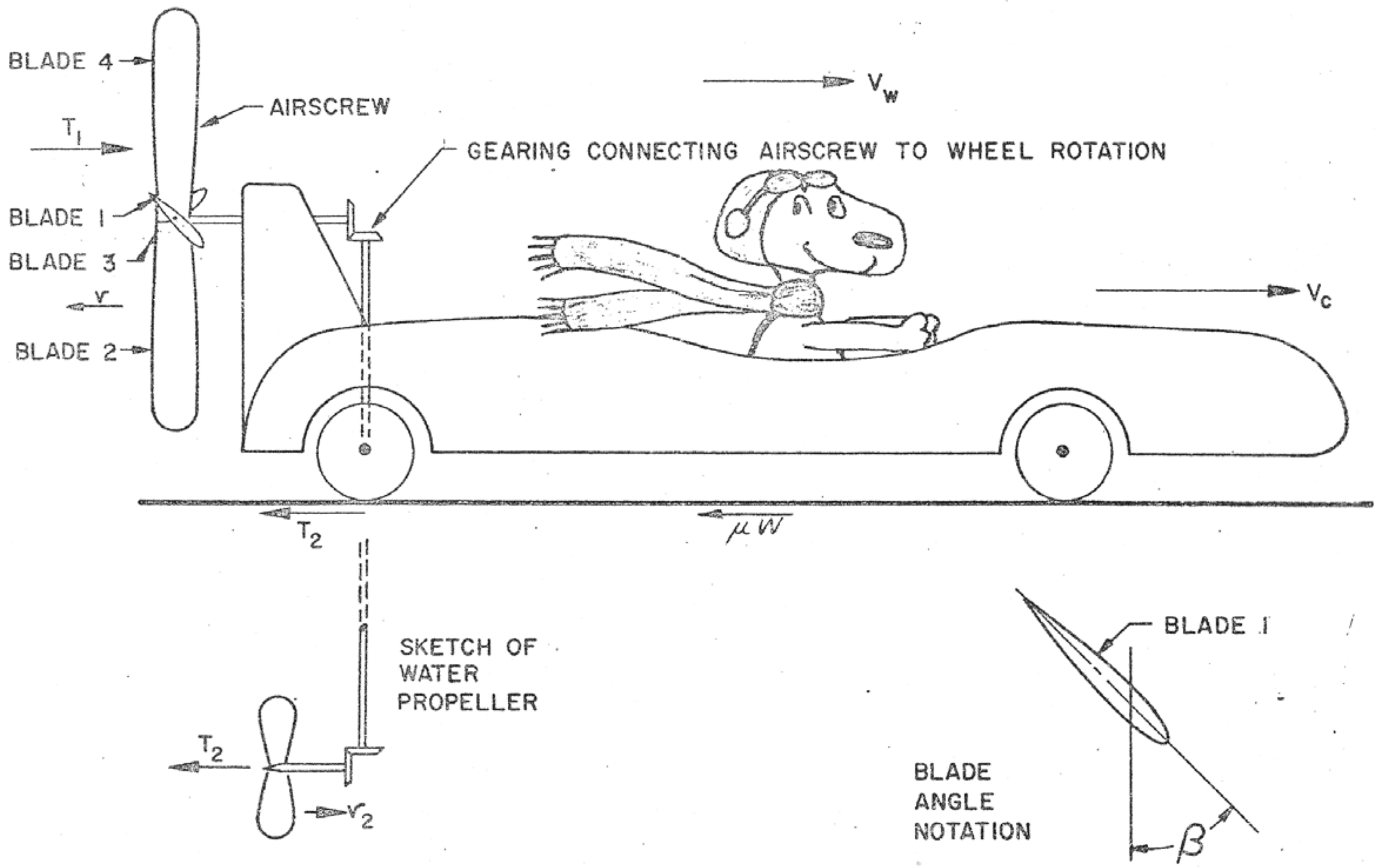

FIGURE 1

VEHICLE SCHEMATIC DIAGRAM

apparent force on the vehicle as a result of acceleration and/or slope of the roadway. With these definitions and sign conventions the equation of motion in the wind direction is

$$T_1 = T_2 + \mu W \tag{3}$$

Hence, the acceleration of the vehicle on a level roadway is given by $\mu_3 g$, where g is the acceleration of gravity. Equivalently, it is convenient to refer to μ_3 as the number of "g's" of vehicle acceleration or simply as the acceleration parameter. In discussing vehicle performance it is also helpful to refer to μ as a vehicle acceleration parameter, since the parameters μ_1 and μ_2 are often small and can be computed easily for most practical cases without the controversy that surrounds the physical problem of computing μ, which is the main objective in writing this paper.

The speed ratio V_c/V_w will be denoted by n. Therefore, in order to go faster than the wind a vehicle must be designed so that μ and μ_3 are larger than zero for speeds up to $n = 1$.

Propeller Theory

The propeller theory needed here for an understanding of the problem is based on simple momentum concepts, as have been discussed in sufficient detail by Glauert [1] and by Prandtl [2]. Other more complex theories have been developed; these ideas apply mainly to lightly loaded propellers and will not be considered here. The present application involves both light and heavy loadings of the propeller, and momentum concepts are adequate for clarifying the physical principles that apply. The flow is taken to be inviscid and incompressible.

Figure 2 illustrates the flow through the propeller or airscrew disk for a wide range of vehicle conditions. For small or negative

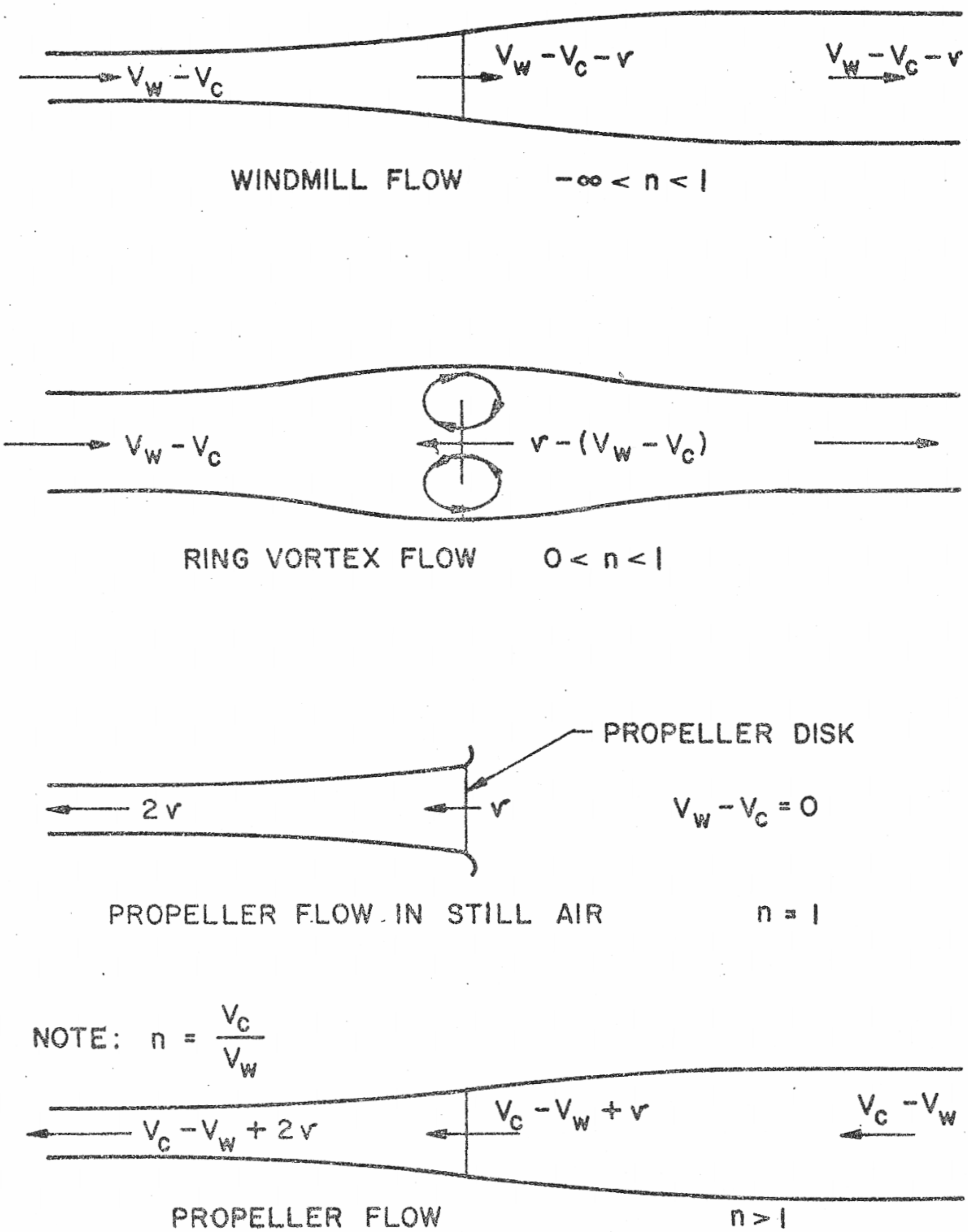

FIGURE 2

DIAGRAMS OF THE FLOW PAST THE AIRSCREW FOR DIFFERENT CONDITIONS. FLOW SPEEDS ARE MEASURED WITH RESPECT TO THE PROPELLER PLANE vehicle speeds where $V_c < V_w$, n will be in the range of $-\infty$ to $+1$. Then, if the propeller is used as a windmill, the stream tube which encloses the propeller tips is illustrated at the top of Figure 2. As with each sketch in Figure 2, the stream tube is drawn in the frame of reference of the propeller disk, and the velocities are taken with respect to the disk. The flow passes through the propeller from left to right with a speed loss v at the propeller disk and a loss of $2v$ far to the right of the disk, as related by momentum theory. Therefore, the mass flow through the disk is

$$\dot{m} = \rho_1 (V + v) A_1 \tag{4}$$

where

$$V = V_c - V_w \tag{5}$$

where ρ_1 is the air density, A_1 is the disk area, and $\dot{m}$ is understood to be positive for flow passing from right to left through the disk, and vice versa.

The above sign convention is chosen so that $\dot{m}$ will be positive for the case of the airscrew acting as a propeller, and negative when acting as a windmill. The airscrew or propeller, as illustrated in Figure 1, is mounted on the vehicle and the "forward" direction, $V_c > 0$, is taken to be toward the right in both Figures 1 and 2. The velocity v induced at the propeller plane is defined as positive when the air is accelerated to the left, which results in a positive thrust T_1 for positive values of v. In general, the thrust is

$$T_1 = 2s\rho_1 A_1 v(V + v) \tag{6}$$

where s is either $+1$ or -1 depending on whether the airscrew is acting as a propeller or windmill, respectively. For the windmill case $2v$ can never be larger than $(V_w - V_c)$, since then the stream tube shown in Figure 2 does not make physical sense.

Equations (4) and (6) use the momentum concept of a uniform flow through the airscrew disk. In actual practice such uniformity is not attained although it may be more closely approximated by increasing the number of airscrew blades. For the ideal condition of a uniform v and no other losses, the power required to drive the airscrew is

$$P_{1_{ideal}} = T_1(V + v) \tag{7}$$

and the ideal efficiency is

$$\eta_{ideal} = \frac{V}{V + v} \tag{8}$$

In an actual propeller the efficiency is about 85 percent of the ideal efficiency under ordinary working conditions [1]. Hence, the actual power required to drive the airscrew is

$$P_1 = \frac{T_1(V + v)}{\eta_1} \tag{9}$$

where η_1 is the result of the above-mentioned nonuniformity, kinetic energy lost to rotation of the slipstream, and energy lost because of blade frictional drag [1]. Equation (9) applies to either the windmill or the propeller case; for the former η_1 will be about $1/0.85$, as P_1 is then negative and more than the ideal P_1. In either case T_1 will be positive, but both P_1 and $(V + v)$ will be negative in the windmill case and positive in the propeller case.

The propeller case is illustrated at the bottom of Figure 2. Here $V_c > V_w$ and $n > 1$. The figure illustrates the stream tube passing over the propeller tips.

A special case is that of $V_c = V_w$ and $n = 1$ so that the vehicle is travelling exactly at the wind speed. Then the airscrew can act only as a propeller, and the stream tube formed by the propeller wake is shown in the third sketch of Figure 2.

The second sketch in Figure 2 is used to illustrate the so-called "ring vortex" flow. This has been described by Glauert in References 1 and 3. This case is of interest when V_c is positive but smaller than V_w so that $0 < n < 1$, and when $\dot{m}$ is positive. Because $\dot{m}$ is positive, the mass flux through the airscrew disk is in a direction opposite to the free-stream direction illustrated by $V_w - V_c$ in the sketch. In the ideal inviscid case $\dot{m}$ would form a stream tube running to the left of the propeller. In the actual case viscous forces would tend to destroy such a stream tube, and the result might be the ring vortex sketch given by Glauert and as shown in Figure 2. The sketch shows a ring vortex surrounding the propeller disk. A stream tube passing around the vortex is also shown.

The author has taken the ring vortex experimental data given by Reference 3 and has compared the values of $T_{1_{exp}}$ derived from this data to the T_1 which would be predicted by momentum theory (Equation (6)) as a function of the ratio $[(V + v)/V]$. The ratio $e = T_{1_{exp}}/T_{1_{mom.\,theory}}$ was found to vary from 0.8 to 1.0 to 1.4 as $[(V + v)/V]$ varied from -2.9 to -1.0 to -0.5, respectively. Since this experimental data was rather meager, and since no corresponding data on P_1 was available, and since $\bar{e}$ was found to be near 1.0 for cases of interest, the author elected to use the momentum theory in the calculations which follow.

Calculation of the Vehicle Acceleration

The power P_1 used to drive the airscrew is delivered from the wheels by means of gearing or a transmission. When P_1 is positive this results in a positive value of T_2 which acts to decrease V_c as shown on Figure 1. In general

$$P_1 = T_2 V_c \tag{10}$$

if the value of η_1 given above is adjusted to account for any friction losses in the transmission. From (9) and (10) we obtain

$$T_2 = \frac{T_1(V + v)}{\eta_1 V_c} \tag{11}$$

and by using (3) to eliminate T_2

$$\mu W = T_1 \left[1 - \frac{(V + v)}{\eta_1 V_c}\right] \tag{12}$$

and by using (6)

$$\mu = \left[1 - \frac{(n - 1 + \lambda_1)}{n\eta_1}\right] s\lambda_1(n - 1 + \lambda_1)Q_1 \tag{13}$$

where

$$\lambda_1 = \frac{v}{V_w} \tag{14}$$

and

$$Q_1 = \frac{2\rho_1 A_1 V_w^2}{W} \tag{15}$$

The Mechanics of Accelerating to the Wind Speed and Faster

Equation (13) shows that the acceleration μ is a function only of the dimensionless coefficients n, λ_1, η_1 and Q_1. For any given speed n the efficiency η_1 will be kept as large as practical so that only λ_1 and Q_1 may be arbitrarily adjusted in order to increase the vehicle acceleration.

For each n there exists a unique value of λ_1 for which μ is maximum. Consider the special and idealized case of $n = 1$ and $\eta_1 = 1$. Then $\mu = (1 - \lambda_1)\lambda_1^2 Q_1$ and μ is maximum for $\lambda_1 = 2/3$. If λ_1 were only 1/2, the mass passing through the propeller plane would be reduced in speed from its original speed V_w to zero speed with respect to the ground. (Refer to the third sketch in Figure 2.) Therefore, the vehicle acceleration is obtained by the process of slowing down the wind so that the kinetic energy per unit mass of the affected air is reduced to zero. In the case of $\lambda_1 = 2/3$ the affected air ends up going to the left at a speed of $V_w/3$ so that its final kinetic energy is then more than zero; μ is larger in this case because the propeller handles a larger mass of air than when $\lambda_1 = 1/2$. If λ_1 were increased to 1, the affected air is sent to the left at a speed V_w so that the process removes zero kinetic energy from the air; then μ is 0. These special cases are instructive as to the physical mechanism of propulsion at the wind speed or $n = 1$.

In more practical cases where $\eta_1 \neq 1$ the magnitude of μ may be increased by increasing the ratio Q_1 or A_1/W for a given wind condition. This shows the desirability of having as large a propeller disk area as possible for any given vehicle weight.

A practical case of vehicle operation may be illustrated by taking η_1 to be 0.80, $\lambda_1 = 0.4$, $Q_1 = 1.0$. Then at $n = 1.0$, μ will be 0.08. Since μ_1 can be made as small as 0.03, and since $\mu_2 = 0$ at $n = 1$, we have $\mu_3 = 0.05$. Therefore, the vehicle will have a forward acceleration of about 1.6 ft/sec^2. The case $Q_1 = 1.0$ corresponds to a 300-pound vehicle operating with an airscrew disk area of 200 ft^2 at sea-level conditions in a wind speed of 17.8 ft/sec or about 12 miles per hour, which corresponds to a rather easily-constructed vehicle. If the wind speed were to drop to 7.4 mph, then μ would be zero.

The above case for a wind speed of 12 mph results in $T_1 = 48$ lb, $T_2 = 24$ lb, $v = 7.1$ ft/sec, $P_1 = 426$ ft-lb/sec or 0.78 horsepower, and $\dot{m} = 3.38$ slugs/sec or 108 lb-mass/sec. The reason that the vehicle is accelerating is simply that the propeller is accelerating the air from zero speed with respect to itself to a speed of 7.1 ft/sec with respect to itself so that the ideal work required is 7.1(48) or 340 ft-lb/sec.

For other vehicle speeds in the neighborhood of $n = 1$ the parameters are much the same. For example, at $n = 1.5$ and for $\lambda_1 = 0.4$ then $\mu = 0.09$.

At smaller speeds with n positive but quite a bit less than 1, the airscrew can be operated as a windmill so that T_1 is positive but T_2 is negative. Since μ_2 is also negative, the only retarding force is the rolling friction $\mu_1 W$. In this situation acceleration is not difficult. At higher speeds T_2 becomes zero and then positive as the airscrew becomes a propeller. Typical performance curves for these operations are given in "Vehicle Performance Calculations".

WATER VEHICLE PERFORMANCE

The water vehicle case is somewhat more complex, inasmuch as the wheels in Figure 1 are replaced by a water propeller as shown schematically on the figure. This second propeller generates a thrust T_2 and an induced water velocity v_2, which are shown in their positive sense on Figure 1. Then the analysis proceeds as before except that equation (10) is replaced by

$$P_1 = \eta_2 T_2 (V_c - v_2) \tag{10w}$$

where η_2 is an efficiency factor similar to η_1 for the first propeller. Here η_2 is less than 1 when η_1 is also less than 1; this is the case of the first propeller or airscrew acting as a propeller rather than a windmill. For the windmill case both η_1 and η_2 are greater than 1.

The thrust T_2 is related to the flow conditions by

$$T_2 = 2s\rho_2 A_2 v_2 (V_c - v_2) \tag{6w}$$

where ρ_2 is the water density and A_2 is the area of the second propeller disk. Then the acceleration equation becomes

$$\mu = \left\{1 - \left[\frac{n - 1 + \lambda_1}{\eta_1\eta_2(n - \lambda_2)}\right]\right\} s\lambda_1(n - 1 + \lambda_1)Q_1 \qquad (13w)$$

where

$$\lambda_2 = \frac{v_2}{V_w} \qquad (14w)$$

For the water case the coefficient μ_1 refers to the hull water drag to weight ratio, μ_2 is the air drag coefficient as before, and μ_3 is also the same as before.

The equation for T_2 may also be put in the dimensionless form

$$\frac{T_2}{W} = s\lambda_2(n - \lambda_2)Q_2 \qquad (16w)$$

where

$$Q_2 = \left(\frac{2\rho_2 A_2 V_w^2}{W}\right) = Q_1\left(\frac{\rho_2 A_2}{\rho_1 A_1}\right) \qquad (15w)$$

VEHICLE PERFORMANCE CALCULATIONS

Vehicle performance has been calculated for both a land vehicle and a water vehicle. Since the writer is in the process of testing one such land vehicle, the land vehicle parameters were chosen here to correspond to the vehicle under test. This vehicle, complete with a 170-pound driver, weighs 270 pounds and has a propeller diameter of 15.4 feet. Standard sea level atmospheric conditions and a wind speed of 16 mph or 14 knots was selected for the calculations; this works out to $Q_1 = 1.8$.

For the boating case, the hull drag and weight becomes a significant problem. For the calculations W was taken to be 600 pounds and the airscrew diameter equal to 26.7 feet so that $Q_1 = 2.45$.

The efficiency factors η_1 and η_2 were taken to be 0.85 and 0.80, respectively, for the cases where the airscrew acts as a propeller. For the windmilling cases η_1 and η_2 were taken as 1/0.85 and 1/0.80. Since the water propeller design is compromised by allowances for cavitation, its efficiency was taken to be less than the air propeller. For the same reason, the ratio $(\rho_2 A_2/\rho_1 A_1)$ was taken as 2.0.

The land vehicle rolling friction coefficient μ_1 was taken to be 0.03; the corresponding hull drag coefficient was based on that given in Figure 19 in the article by Davidson [4]. The wind friction coefficient μ_2 was based on the usual aerodynamic drag equation

$$D = \frac{\rho_1}{2} V^2 S$$

where S is the so-called flat plate drag area. This was taken to be 8 ft^2 for the land vehicle and 6 ft^2 for the boating case.

The procedure for calculating μ is not straightforward because λ_1 and λ_2 are unknowns. For the land vehicle at the lower speeds λ_1 was calculated by first assuming a lift coefficient of 1.0 and using the propeller solidity factor of 0.133 and the gearing factor G of 4.0 so that the propeller blade element forces could be integrated to determine an upper limit to T_1. Equation (6) was then used to obtain λ_1. This procedure was modified at the higher speeds because it was found that a decreased λ_1 and lift coefficient resulted in a maximum value for μ.

In the water vehicle case the calculation was complicated by the fact that the equation for T_2, (16w), must be satisfied simultaneously with Equation (13w). Also, T_2 was always related to T_1 and μ by Equation (3). An iteration procedure was used to obtain such solutions.

Figure 3 gives the values obtained for λ_1 and λ_2. The first airscrew was used as a propeller for speeds of $n > 0$ and as a windmill

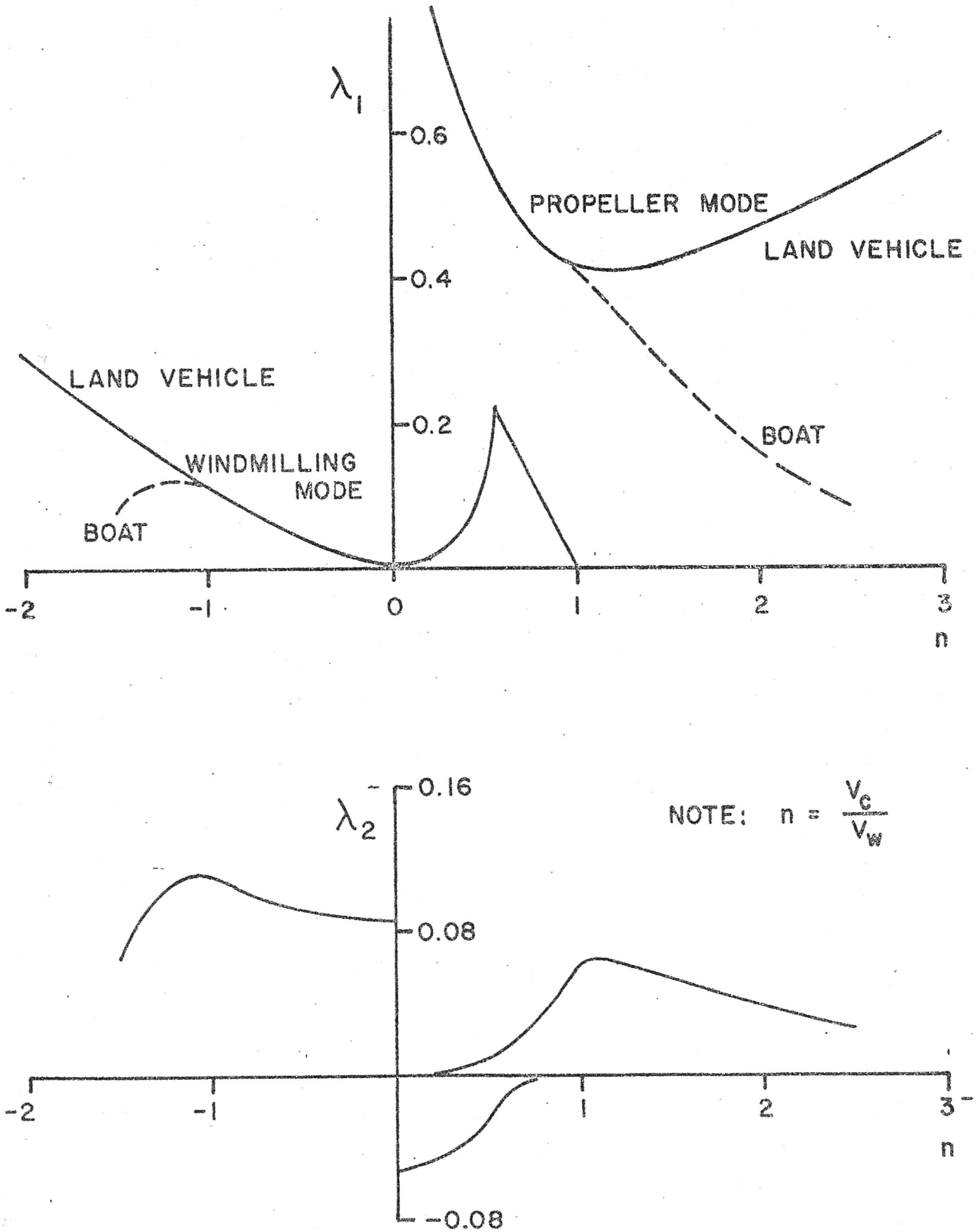

FIGURE 3

AIRSCREW LOADING PARAMETERS λ_1 AN λ_2 USED TO CALCULATE TYPICAL VEHICLE PERFORMANCE for speeds $n < 1$. Hence, the region $0 < n < 1$ offers the possibility of either the propeller or windmilling mode of operation. The sharp break in the windmilling operation curves at $n = 0.55$ resulted from the necessity to keep $2v$ less than $(V_w - V_c)$ for $n > 0.55$.

The resulting vehicle accelerations are shown on Figure 4. For $n < 0$ the sign convention implies that μ should be negative for speed "increases" toward the left or in the weather direction. These negative values have been rectified on Figure 4 by use of the absolute value signs. The curves show that the land vehicle is capable of operating over the range of $n = -2$ to $n = +3$. The crossover point between the windmilling and propeller modes is at $n = 0.6$ for optimum acceleration. The minimum in the acceleration curves at this point is only computational. In actual fact a third mode is possible at this point wherein the propeller is transmitting zero power to the wheels and the vehicle is pushed forward entirely by thrust T_1 with $T_2 = 0$. Other modes with varying degrees of T_1 and T_2 are also possible; these are not discussed further here.

Figure 4 shows that the water vehicle has similar characteristics, but because of the extra losses of a second propeller μ is not as good as for the land vehicle. Also, the hull drag is much more significant than in the land vehicle case, so that μ_3 is reduced. The net result shows the vehicle operable over the range of $-0.85 < n < 1.5$.

For speeds other than $V_w = 14$ knots both μ and μ_2 vary as V_w^2. The same is approximately true for μ_1 in the boating case, so that the value shown for μ_3 also must vary approximately as V_w^2. Therefore, the water vehicle performance shown by Figure 4 is representative for all wind speeds. The same is not true for the land vehicle wherein μ_1 tends to be constant, independent of speed. Therefore, below a certain wind

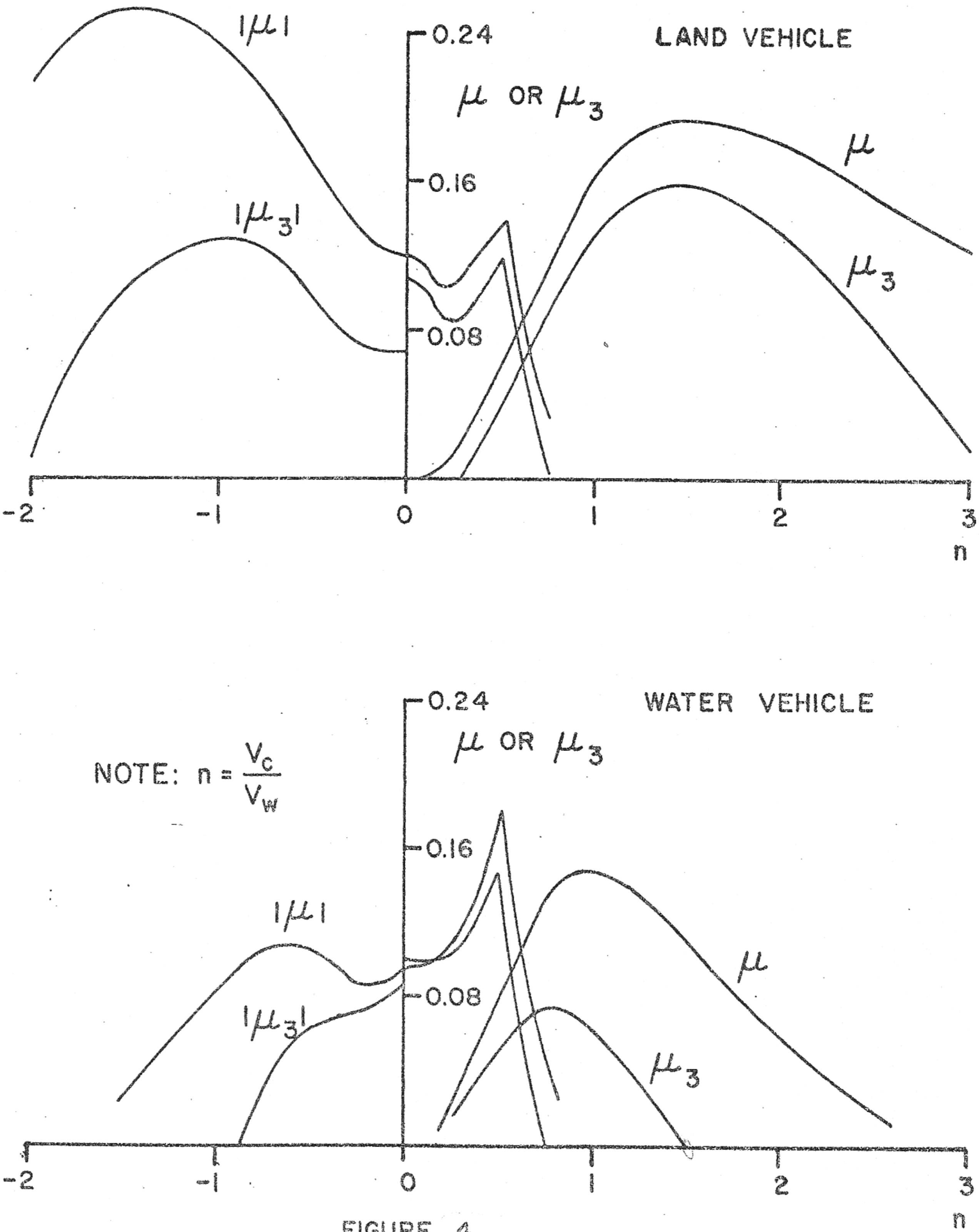

FIGURE 4

VEHICLE ACCELERATION CURVES

speed μ_1 will be so large with respect to μ that the land vehicle may not be able to accelerate over part or all of the speed range. For example, if V_w were reduced to 7 knots, the acceleration μ_3 for the land vehicle would be reduced to approximately zero for the speed range of $-0.5 < n < 0.8$.

The above performance curves were obtained with somewhat arbitrary values of vehicle weight and propeller disk areas. Obviously care is required so that the above ratios of A_1/W may be met in practice, and with large vehicles more optimistic results may be obtained. Also, the values of η_1 and η_2 are perhaps a bit optimistic and the results should be viewed accordingly. Finally, much is yet to be learned about modes of operation so that λ_1 and λ_2 are kept near the optimum values.

LAND VEHICLE TRIAL RUNS

Since 1 February 1969 a few runs of the land vehicle have been made. The vehicle was found operable from the outset, but some mechanical problems have limited the performance. The vehicle uses an eight-foot loop of bicycle chain to transmit power from the propeller shaft to the wheels. The chain has had the inconvenient habit of coming off the sprockets at inopportune moments. However, since chains may be operated at efficiencies of 98 to 99%, the chain was felt to offer the best, as well as one of the simplest, means of power transmission. Work is underway to develop a chain guide to remedy the problem.

The vehicle has been controlled by its front steering wheel and a lever for changing the angle of the two propeller blades. The angle β, as defined on Figure 1, can travel from -130 to +90 degrees when measured at a station 70% of the distance to the propeller tip. The blades have a moderate amount of twist so that the twist is about right for $n = 1.5$ and $\beta \approx 20$ degrees. G is 4.0.

When downwind operation is desired, the blades are placed at $\beta = -90^{\circ}$ and then rotated forward until the wind generates sufficient airscrew torque to start the machine rolling. As the vehicle speed increases β is increased for further acceleration until β is in the neighborhood of 20° at $n = 1$. Further increases in β increase the speed, and a decrease in β will decrease the speed and bring the vehicle to a rather quick stop. The only difficulty in this regard has been when the chain became disengaged so that the braking action to the wheels was lost. The wind speeds encountered have been about 10 mph and maximum speed travelled has been about 15 mph, at which point the chain became disengaged. The amount by which the vehicle exceeded the wind speed was estimated from the rearward deflection of a foot-long tuft located about 12 feet forward of the propeller plane.

Sustained runs of the order of 40 seconds have been made in a wind of about 12 mph with the vehicle speed estimated to be about 2 mph faster than the wind. More such runs are planned for the future, as the runs to date have been too short to do much experimentation such as recording the effects of blade angles on speed.

Since the blades were twisted for $n = 1.5$, the twist turns out to be in the wrong direction for the windmilling required in the speed range $0 < n < 0.5$. This limitation has not caused any difficulty in operating the vehicle.

On going windward the vehicle has been limited in speed to about 6 mph by the blade angle stop at $\beta = 130^{\circ}$. The general procedure has been to start at zero speed with β at -90° and gradually decrease β to -130° as speed is attained.

CONCLUSIONS

1. The general concept of a wind powered vehicle capable of traveling straight downwind at speeds exceeding the wind speed has been demonstrated for both ground and water vehicles.

2. The practicability of traveling directly to weather in such a vehicle has also been demonstrated.

3. The natural extension of the above is to construct a vehicle capable of traveling in any desired direction.

4. The performance limits of such vehicles must await a development program to test vehicle construction methods, operational techniques, and efficiency factors.

ACKNOWLEDGMENTS

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5. APPENDIX

5.1 BIOGRAPHY

Walter John Johnson, 34 years old, the happy son of a baker, received his bachelor of science in electrical engineering in 1966 at the University of Washington.

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(2) Two articles on the energy transfer scheme appear in the Proceedings of the First AIAA Symposium on Sailing, April, 1969. A.G. Hammitt, Optimum Wind Propulsion; and Andrew B. Bauer, Faster than the Wind. (Someone should work up a proof that the opposing force vector vehicle is more efficient than the energy transfer vehicle.)

(3) C.A. Marchaj; Sailing Theory and Practice: This is probably the best book on sailing available; however, like most researchers of the species, he is short on hull data and total performance.

(4) S. F. Hoerner; Fluid-Dynamic Drag: This book is a "must" for anyone in aerodynamics and hydrodynamics research.

5.3 SYMBOLS

A_A = area of the sail

A_H = area of the hull

C_{LA} = side force coefficient for sail

C_{LH} = side force coefficient for hull

P = performance limit

$S/\overset{A}{D}$ = side force-to-drag on the airfoil

$S/\overset{H}{D}$ = side force-to-drag on the hull

V_B = velocity of the boat

V_W = velocity of the wind

θ = theta = angle between wind and boat (direction of travel)

δ = delta = $\tan^{-1} S/D$ for airfoil

ϕ = phi = $\tan^{-1} S/D$ for hull